DOI: https://doi.org/10.26642/ten-2025-1(95)-304-311
UDC 621.317

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Application of wavelet transform for image compression in information-measuring systems for medical use

In modern information and measuring systems (IMS), the most informative type of data is images. These can be images that characterize the condition of patients, and the system will have medical applications. Images can be formed in the visible range of electromagnetic radiation waves, or can be the result of recording other types of radiation. High information content, as an important advantage of images, leads to the need to transmit and store a large amount of digital data. Therefore, for effective use of computerized IMS resources, it is necessary to reduce this volume by compression. The required number of compression times is from several tens to hundreds of times.

The article considers the theoretical foundations of the direct and inverse wavelet transform and its application in image compression procedures for medical applications, which occurs with some loss of information. By controlling the compression procedure and choosing its parameters, it is possible to ensure an acceptable error in image recovery.

It is determined that compression based on the selected wavelet type is possible when it is orthogonal and, accordingly, there is an inverse wavelet transform. Also, to increase the speed of compression procedures, it is proposed to divide the wavelet transform into two one-dimensional procedures applied to rows and columns of the image.

An example of the influence of wavelet compression in the JPEG-2000 format on the accuracy of transmitting information about geometric parameters and coordinates of contour points of objects in medical applications of images in IBS is considered.

Keywords: *information-measuring system; medical diagnostic images; video images; compression with partial information loss; wavelet transform.*

Relevance of the topic. Digital video images occupy an increasingly significant portion of the information world, driving continuous interest in improving data compression algorithms for video images. Compression is crucial for both transmission speed and storage efficiency. The growing volume of medical data necessitates effective compression methods for storage and further processing. Compression and encoding algorithms, such as wavelet transforms for medical images, help reduce data volume without losing critical information. Wavelet-based JPEG 2000 compression can achieve 20 % better efficiency compared to previous JPEG DCT methods [13]. JPEG 2000 also leverages the progressive transmission property of wavelets, allowing users to access images with progressive quality in terms of resolution and color depth.

Analysis of recent research and publications referenced by the authors. Digital image processing methods are discussed in the works of R.Gonzalez, R.Woods, Wilhelm Burger, B.Jeahne, G.Gimel'farb, P.Delmas, N.Kehtarnavaz, M.Gamadia [1–7], and methods of image compression are reviewed in [8–17].

The objective of the article is to analyse the wavelet transform for video image compression with partial information loss in medical information-measuring systems, aiming to reduce data volume while preserving quality.

Presentation of the main material. There are two main ways to compress a dataset represented as a list of numbers: either reduce the list's length or reduce the average number of bits required to represent each number. Comprehensive compression schemes use both approaches [9].

Transform methods aim to convert image data into a form where it is easier to determine which parts can be discarded with minimal quality loss. This allows significant information removal while maintaining acceptable image quality. For Fourier transforms, this typically involves discarding high-frequency components. For wavelets, it's the components corresponding to fine details. Fractal methods aim to represent image data compactly through inherent self-similarity.

The result of algorithmic encoding can be further compressed by compactly representing the encoded numbers. Quantization can reduce both the number of values in the list and the number of bits needed per value. Digital images are inherently quantized before encoding.

During decoding, the original image is reconstructed from the encoded data. In transform methods, this involves applying the inverse transform. Post-processing may be applied to improve the quality of the decoded image.

Another method to reduce data volume is quantization. Scalar quantization reduces precision to a fixed level, which can be uniform or non-uniform. Uniform quantization distributes quantization levels evenly across the value range and works best when the values are uniformly distributed [1]. For non-uniform distributions, denser quantization levels may be beneficial in areas with higher value concentrations.

Decimation, a type of non-uniform quantization, involves zeroing out a portion of values. One approach in wavelet image compression is to zero out, say, 90 % of wavelet coefficients. The remaining values are simply stored. The number of quantization levels is equal to zero plus the number of unique remaining values.

Vector quantization replaces a set of values with a single quantized value and is useful in encoding color images, where each pixel is typically represented by a triplet of values.

Wavelet Transform

Assume a sequence of 2^n points $\{x_1, x_2, ..., x_{2^n}\}$ for some integer $n>0$. This sequence can be identified with V^n [16]:

$$f(t) = x_1\phi_{n,0}(t) + ... + x_{2^n}\phi_{n,2^n-1}(t).,\qquad (1)$$

The first step in computing the wavelet transform of the sequence $\{x_1, x_2, ..., x_{2^n}\}$ will be $f(t)$ decomposition in an alternative basis of the space V^n, of the form $\left\{\phi_{k,0}, ..., \phi_{k,2^k-1}, \psi_{k,0}, ..., \psi_{k,2^k-1}\right\}$, half of which consists of wavelets [16]:

$$\begin{aligned} f(t) &= a_{n-1,0}\phi_{n-1,0}(t) + ... + a_{n-1,2^{n-1}-1}\phi_{n-1,2^{n-1}-1}(t) + \\ &+ d_{n-1,0}\psi_{n-1,0}(t) + ... + d_{n-1,2^{n-1}-1}\psi_{n-1,2^{n-1}-1}(t) \end{aligned},\qquad (2)$$

The $d_{n-1}, ..., d_{n-1,2^{n-1}-1}$ coefficients of the basis wavelet functions make up half of the wavelet transform coefficients, so these values must be stored. The next step in the transformation process is to apply a basis transformation to the remaining terms of equality (2):

$$g_{n-1}(t) = a_{n-1,0}\phi_{n-1,0}(t) + ... + a_{n-1,2^{n-1}-1}\phi_{n-1,2^{n-1}-1}(t),\qquad (3)$$

Thus, g_{n-1} is an element of V^{n-1}, and therefore can be decomposed using an alternative basis consisting of scaling functions $\phi_{n-2,j}$ and wavelets $\psi_{n-2,j}$.

To obtain the coefficients of equality (2) from the coefficients of equality (1), we use orthogonality. Each scaling function $\phi_{n-1,j}$ is orthogonal to every $\phi_{n-1,k}$, as well as to all $\psi_{n-1,j}$. Similarly, each wavelet $\psi_{n-1,j}$ is orthogonal to other wavelets $\psi_{n-1,k}$ and to all scaling functions $\phi_{n-1,j}$ .Each $\phi_{n-1,j}$ and each $\psi_{n-1,j}$ is normalized due to the identities $\phi_{k,j}(t) \equiv \sqrt{2^k}\phi(2^k t - j),\ j = 0, ..., 2^k - 1$ and $\psi_{k,j}(t) \equiv \sqrt{2^k}\psi(2^k t - j),\ j = 0, ..., 2^k - 1$. To take advantage of this orthogonality and normalization, we multiply both sides of (2) by $\phi_{n-1,j}(t)$ and integrate over t from 0 to 1. As a result, we obtain [16]:

$$\int_0^1 f(t)\phi_{n-1,j}(t)dt = a_{n-1,j}\ ,\qquad (4)$$

Due to orthogonality, only one term remains on the right-hand side of equation (4), and normalization results in the absence of a coefficient before $a_{n-1,j}$. Now, we substitute the right-hand side of equation (1) in place of $f(t)$ in equation (4). For example, when j=0, the left-hand side of equation (4) will be equal to:

$$\int_0^{1/2^n} x_1\sqrt{2^n}\sqrt{2^{n-1}}dt + \int_{1/2^n}^{2/2^n} x_2\sqrt{2^n}\sqrt{2^{n-1}}dt = (x_1 + x_2)\left(1/\sqrt{2}\right)2^n\left(1/2^n\right) = \frac{x_1 + x_2}{\sqrt{2}}\ ,\qquad (5)$$

Combining (4) and (5) for j=0, we obtain:

$$a_{n-1,0} = \frac{x_1 + x_2}{\sqrt{2}}\ ,\qquad (6)$$

The square root in the coefficient in (6) appears due to normalization. The other coefficients $a_{n-1,j}$, $j = 1, ..., 2^{n-1} - 1$ are calculated similarly. Thus,

$$a_{n-1,j} = \frac{x_{2j+1} + x_{2j+2}}{\sqrt{2}}, \quad j = 0,...,2^{n-1} - 1, \tag{7}$$

Similarly, using the properties of orthogonality and normalization of $\psi_{n-1,j}$ functions, the $d_{n-1,j}$ coefficients can be calculated using the following formula [16]:

$$d_{n-1,j} = \frac{x_{2j+1} - x_{2j+2}}{\sqrt{2}}, \quad j = 0,...,2^{n-1} - 1, \tag{8}$$

Equations (7) and (8) can be conveniently represented in the form of a single matrix equation:

$$\begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & \cdot & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & \cdot & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix} \begin{bmatrix} x_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ x_{2^n} \end{bmatrix} = \cdot \begin{bmatrix} a_{n-1,0} \\ \cdot \\ \cdot \\ a_{n-1,2^{n-1}-1} \\ d_{n-1,0} \\ \cdot \\ \cdot \\ d_{n-1,2^{n-1}-1} \end{bmatrix}, \tag{9}$$

The matrix in (9) is a square matrix with 2^n rows and 2^n columns. Let us define:

$$\mathbf{A}_n = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & \cdot & 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \tag{10}$$

i

$$\mathbf{D}_n = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & \cdot & \cdot & \cdot & \cdot & 0 \\ 0 & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & \cdot & 0 & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{bmatrix}, \tag{11}$$

where $\mathbf{A}_n$ and $\mathbf{D}_n$ are matrices of size $2^{n-1} \times 2^n$, and $\mathbf{A}_n$ can be considered as an averaging operator, while $\mathbf{D}_n$: $R^{2^n} \to R^{2^{n-1}}$ is a difference operator. Let us introduce the vector notation [16]:

$$\mathbf{x} = \begin{bmatrix} x_1 \\ \cdot \\ \cdot \\ x_{2^n} \end{bmatrix}, \ \mathbf{a}_{n-1} = \begin{bmatrix} a_{n-1,0} \\ \cdot \\ \cdot \\ a_{n-1,2^{n-1}-1} \end{bmatrix}, \ \mathbf{d}_{n-1} = \begin{bmatrix} d_{n-1,0} \\ \cdot \\ \cdot \\ d_{n-1,2^{n-1}-1} \end{bmatrix}. \tag{12}$$

where $\mathbf{x}$ is a column vector with 2^n elements, $\mathbf{a}_{n-1}$ and $\mathbf{d}_{n-1}$ are column vectors with 2^{n-1} elements. Equation (24) can be rewritten as:

$$\begin{bmatrix} \mathbf{A}_n \\ \hdashline \mathbf{D}_n \end{bmatrix} \mathbf{x} = \begin{bmatrix} \mathbf{a}_{n-1} \\ \hdashline \mathbf{d}_{n-1} \end{bmatrix}, \tag{13}$$

The matrix on the left-hand side of equation (28) is a single matrix $2^n \times 2^n$, and the vector on the right-hand side is a single column vector $2^n \times 1$. At each step of the wavelet transform process, the detailing coefficients are stored, and the averaging coefficients are processed. In the considered case, the wavelet transform will have 2^n components. Half of them are obtained from equation (13) as detailing coefficients in $\mathbf{d}_{n-1}$. These coefficients are stored as half of the wavelet transform. The next step of the wavelet transform involves applying averaging and differencing operations to $\mathbf{a}_{n-1}$ at the next, lower level of resolution [16]:

$$\begin{bmatrix} \mathbf{A}_{n-1} \\ \hdashline \mathbf{D}_{n-1} \end{bmatrix} \mathbf{a}_{n-1} = \begin{bmatrix} \mathbf{a}_{n-2} \\ \hdashline \mathbf{d}_{n-2} \end{bmatrix}, \tag{14}$$

Here, $\mathbf{A}_{n-1}$ and $\mathbf{D}_{n-1}$ are $2^{n-2} \times 2^{n-1}$ matrices of the form (10) and (11), $\mathbf{a}_{n-2}$ and $\mathbf{d}_{n-2}$ are column vectors of dimension 2^{n-2}. To construct part of the wavelet transform, we store $\mathbf{d}_{n-2}$ together with $\mathbf{d}_{n-1}$. This process continues by applying averaging and differencing operations to $\mathbf{a}_k$ and storing the resulting detailing coefficients as part of the wavelet transform. In the final step, the average value $\mathbf{a}_0$ is stored, which is a single-component vector (i.e., a scalar) with the sole element a_{00}. The resulting wavelet transform, which can be represented as a single column vector with $1+1+2+...+2^{n-1}=2^n$ elements, will take the following form [16]:

$$\begin{bmatrix} \mathbf{a_0} \\ \mathbf{d_0} \\ \mathbf{d_1} \\ \cdot \\ \cdot \\ \cdot \\ \mathbf{d_{n-1}} \end{bmatrix}, \tag{15}$$

Inverse Wavelet Transform

In order for the wavelet transform to be used in applications such as video compression, it must be reversible. This means that, for a given wavelet transformation in the form of (30), we should be able to restore the original sequence $\{x_1, x_2, ..., x_{2^n}\}$ from which the transformation was obtained. The step of the wavelet transformation, which involves transitioning from level k resolution to level k-1, is as follows [16]:

$$\begin{bmatrix} \mathbf{A}_{k} \\ \hdashline \mathbf{D}_{k} \end{bmatrix} \mathbf{a}_{k} = \begin{bmatrix} \mathbf{a}_{k-1} \\ \hdashline \mathbf{d}_{k-1} \end{bmatrix}, \tag{16}$$

Therefore, we get:

$$\begin{aligned} (a_{k,0} + a_{k,1})/\sqrt{2} &= a_{k-1,0} \\ (a_{k,0} - a_{k,1})/\sqrt{2} &= d_{k-1,0} \end{aligned}, \tag{17}$$

Note that (31) and (32) can be written as a pair of matrix equations:

$$\begin{aligned} \mathbf{A}_k \mathbf{a}_k &= \mathbf{a}_{k-\mathbf{1}} \\ \mathbf{D}_k \mathbf{a}_k &= \mathbf{d}_{k-1} \end{aligned}, \tag{18}$$

for each k=1,…,n.

From (17) using the known expressions for the lower level of resolution $a_{k-1,0}$ and $d_{k-1,0}$, we obtain expressions for the higher level of resolution $a_{k,0}$, $a_{k,1}$:

$$a_{k,0} = (a_{k-1,0} + d_{k-1,0})/\sqrt{2}$$

$$a_{k,1} = (a_{k-1,0} - d_{k-1,0})/\sqrt{2}$$

Similarly, for j=0,…,2^k-1,

$$\begin{aligned} a_{k,2j} &= (a_{k-1,j} + d_{k-1,j})/\sqrt{2} \\ a_{k,2j+1} &= (a_{k-1,j} - d_{k-1,j})/\sqrt{2} \end{aligned}, \tag{19}$$

From the point of view of linear algebra, this is the solution of equation (16) with respect to the vector $\mathbf{a}_k$ by inverting the matrix of the left-hand side of this equation. The operation performed is [16]:

$$\mathbf{a}_k = \begin{bmatrix} \mathbf{A}_k \\ \hdashline \mathbf{D}_k \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{a}_{k-1} \\ \hdashline \mathbf{d}_{k-1} \end{bmatrix}, \tag{20}$$

Thus, the inverse matrix in (20) exists and can be found in (19). In matrix form, this inversion looks like

$$\begin{bmatrix} \mathbf{A}_k \\ \hdashline \mathbf{D}_k \end{bmatrix}^{-1} = \begin{bmatrix} \frac{1}{\sqrt{2}} & 0 & \cdot & \cdot & \cdot & 0 & \frac{1}{\sqrt{2}} & 0 & \cdot & \cdot & \cdot & 0 \\ \frac{1}{\sqrt{2}} & 0 & \cdot & \cdot & \cdot & 0 & \frac{-1}{\sqrt{2}} & 0 & \cdot & \cdot & \cdot & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & \cdot & \cdot & \cdot & 0 & \frac{1}{\sqrt{2}} & 0 & \cdot & \cdot & 0 \\ 0 & \frac{1}{\sqrt{2}} & 0 & \cdot & \cdot & \cdot & 0 & \frac{-1}{\sqrt{2}} & 0 & \cdot & \cdot & 0 \\ 0 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ 0 & \cdot & \cdot & \cdot & 0 & \frac{1}{\sqrt{2}} & 0 & \cdot & \cdot & \cdot & 0 & \frac{1}{\sqrt{2}} \\ 0 & \cdot & \cdot & \cdot & 0 & \frac{1}{\sqrt{2}} & 0 & \cdot & \cdot & \cdot & 0 & \frac{-1}{\sqrt{2}} \end{bmatrix}, \tag{21}$$

Comparison with (9) shows that this inverse matrix is simply the transpose of the direct transformation matrix. In fact, the first 2^{k-1} columns of the matrix in (21) are exactly the transpose of $\mathbf{A}_k$ matrix, and the last 2^{k-1} columns $\mathbf{D}_k$. This convenient property arises from the orthogonality and normality of the scale basis functions and wavelet functions.

Thus, the transformation in (21) can be written as [16]:

$$\begin{bmatrix} \mathbf{A}_k \\ \hdashline \mathbf{D}_k \end{bmatrix}^{-1} = \begin{bmatrix} \mathbf{A}_k^* & \vdots & \mathbf{D}_k^* \end{bmatrix}, \tag{22}$$

where * denotes matrix transposition.

Equation (20) can be rewritten in the form:

$$\mathbf{a}_k = \begin{bmatrix} \mathbf{A}_k \\ \hdashline \mathbf{D}_k \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{a}_{k-1} \\ \hdashline \mathbf{d}_{k-1} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_k^* & \vdots & \mathbf{D}_k^* \end{bmatrix} \begin{bmatrix} \mathbf{a}_{k-1} \\ \hdashline \mathbf{d}_{k-1} \end{bmatrix} = \mathbf{A}_k^* \mathbf{a}_{k-\mathbf{1}} + \mathbf{D}_k^* \mathbf{d}_{k-1}., \tag{23}$$

Equation (23) gives a practical formula for obtaining $\mathbf{a}_k$ from $\mathbf{a}_{k-1}$ i $\mathbf{d}_{k-1}$: we apply $\mathbf{A}_k^*$ to $\mathbf{a}_{k-1}$ and $\mathbf{D}_k^*$ to $\mathbf{d}_{k-1}$ and add the results. Comparison with (18) shows that the following is true:

$$\mathbf{a}_k = \mathbf{A}_k^* \mathbf{A}_k \mathbf{a}_k + \mathbf{D}_k^* \mathbf{D}_k \mathbf{a}_k, \tag{24}$$

In fact, it is possible to directly prove the truth of the following relation:

$$\mathbf{A}_k^* \mathbf{A}_k + \mathbf{D}_k^* \mathbf{D}_k = \mathbf{I}_{2^k}, \tag{25}$$

where $\mathbf{I}_{2^k}$ is the $2^k \times 2^k$ identity matrix. The following relation also holds:

$$\begin{aligned} \mathbf{A}_k \mathbf{A}_k^* &= \mathbf{I}_{2^{k-1}} \\ \mathbf{D}_k \mathbf{D}_k^* &= \mathbf{I}_{2^{k-1}}. \end{aligned}, \tag{26}$$

Two-Dimensional Wavelet Transform

It is possible to extend the idea of wavelet transformation to larger sequences. The first approach involves first transforming the rows of the image, and then transforming the columns of the image that already have transformed rows. This is easily implemented in software, as the same one-dimensional transformation can be used for both the rows and the columns of the image [1].

To see how this procedure works, let us consider a two-dimensional analog of a simple 4-element sequence $\{x_1, x_2, x_3, x_4\}$. Assume we have a 4×4 image [16]:

$$\begin{bmatrix} x_{1,1} & x_{1,2} & x_{1,3} & x_{1,4} \\ x_{2,1} & x_{2,2} & x_{2,3} & x_{2,4} \\ x_{3,1} & x_{3,2} & x_{3,3} & x_{3,4} \\ x_{4,1} & x_{4,2} & x_{4,3} & x_{4,4} \end{bmatrix}, \quad (27)$$

which can be represented as a function defined on the unit square $\lfloor 0,1 \rfloor \times \lfloor 0,1 \rfloor$:

$$f(x,y) = \sum_{i=1}^{4} \sum_{j=1}^{4} x_{i,j} X_{I_i \times I_j}(x,y), \quad (28)$$

Here

$$I_i \times I_j \equiv \left[\frac{i-1}{4}, \frac{i}{4}\right) \times \left[\frac{j-1}{4}, \frac{j}{4}\right) = \left\{(x,y): x \in \left[\frac{i-1}{4}, \frac{i}{4}\right) \quad i \quad y \in \left[\frac{j-1}{4}, \frac{j}{4}\right)\right\}$$

$$\begin{aligned} X_{I_i \times I_j}(x,y) &= \begin{cases} 1 & if \ (x,y) \in I_i \times I_j \\ 0 & else \end{cases} \\ &= X_{I_i}(x) X_{I_j}(y), \\ &= \phi_{2,i-1}(x) \phi_{2,j-1}(y) \end{aligned} \quad (29)$$

Substituting (29) into (28), we obtain

$$\begin{aligned} f(x,y) &= \sum_{i=1}^{4} \sum_{j=1}^{4} x_{i,j} \phi_{2,j-1}(x) \phi_{2,j-1}(y) = \sum_{i=1}^{4} \left\{ \sum_{j=1}^{4} x_{i,j} \phi_{2,j-1}(y) \right\} \phi_{2,j-1}(x) = \\ &= \sum_{i=1}^{4} \tilde{x}_i(y) \phi_{2,i-1}(x) \end{aligned} \quad (30)$$

where

$$\tilde{x}_i(y) = \sum_{j=1}^{4} x_{i,j} \phi_{2,j-1}(y), \quad (31)$$

The one-dimensional wavelet transform can be applied to this equation for each i=1,...,4 in (31). As a result, we obtain a new set of equations for $\tilde{x}_i(y)$ i=1,…,4, with coefficients that are the result of the wavelet transform of the sequence $\{x_{i,1},..., x_{i,4}\}$. Thus, we obtain the following [16]:

$$\tilde{x}_i(y) = a_{0,0}^i \phi_{0,0}(y) + d_{0,0}^i \psi_{0,0}(y) + d_{1,0}^i \psi_{1,0}(y) + d_{1,1}^i \psi_{1,1}(y), \quad (32)$$

For each I=1,...,4. This is equivalent to applying the one-dimensional wavelet transform to each row of the original image (27). Now, let us substitute (32) back into (30) and rearrange the terms to obtain [16]:

$$\begin{aligned} f(x,y) = &\left\{ \sum_{i=1}^{4} a_{0,0}^i \phi_{2,i-1}(x) \right\} \phi_{0,0}(y) + \left\{ \sum_{i=1}^{4} d_{0,0}^i \phi_{2,i-1}(x) \right\} \psi_{0,0}(y) + \\ &\left\{ \sum_{i=1}^{4} d_{1,0}^i \phi_{2,i-1}(x) \right\} \psi_{1,0}(y) + \left\{ \sum_{i=1}^{4} d_{1,1}^i \phi_{2,i-1}(x) \right\} \psi_{1,1}(y) \end{aligned}, \quad (33)$$

Each of the sums in parentheses in (33) is again very similar to the equation $f(t) = x_1 \phi_{2,0}(t) + x_2 \phi_{2,1}(t) + x_3 \phi_{2,2}(t) + x_4 \phi_{2,3}(t)$, and therefore the one-dimensional wavelet transform can be applied to each of these expressions. This is equivalent to applying the one-dimensional wavelet transform to each column of the original image (27).

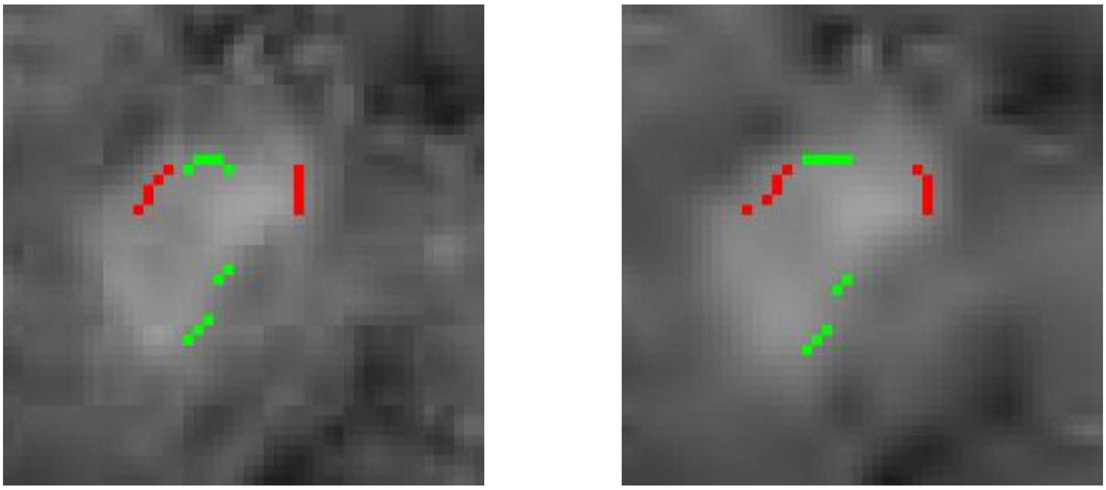
Thus, one way to obtain a two-dimensional wavelet transform for an image of size $2^n \times 2^n$ is to first apply the one-dimensional wavelet transform to each of the 2^n rows, and then apply the one-dimensional wavelet transform to each of the 2^n columns. This is not the only way to obtain the two-dimensional wavelet transform. This approach has the obvious advantage of implementation, as it requires no modification of the already existing one-dimensional transform. It is very efficient for video image compression purposes [16].

Errors in Object Coordinates on Reconstructed Video Images

One of the most important methods for reducing the volume of video data (i.e., its compression) is the selection of optimal resolution parameters for brightness, color, and spatial features of video image elements. Exceeding these parameters beyond values noticeable by the human eye will only increase the volume of the video data without improving its visual quality. In contrast, when measuring the geometric parameters of objects, it is necessary to choose parameter values that ensure the required accuracy of the measurement video information [11].

The main parameter of the compression algorithm is the quantization table of frequency coefficients obtained through wavelet transformation. By modifying the elements of this table, one can change the compression ratio. Moreover, it also affects the accuracy of video image reconstruction after compression.

The assessment of coordinate errors is based on the analysis of video image segments whose discrete points share the same features. These segments correspond to the measured objects and are obtained by segmenting test video images. The segmentation feature was the brightness value of discrete points in the digital video image. Figure 1 shows the displacement of contour points during the compression of the halftone equivalent of the video image. For video images, points whose brightness lies within the range {128, 255} of discrete video signal levels were considered bright, while others in the {0, 127} range were considered dark.



a) *b)*

Fig. 1. Displacement of contour points during compression of the halftone equivalent of the video image: a) original video image, b) reconstructed video image after 50x compression

The measure of coordinate distortion is a quantity determined by the formula [11]:

$$\Delta_{coord.} = \sqrt{\frac{1}{L}\sum_{i=1}^{L}\left(R_{compr.}(i) - R_{orig.}(i)\right)^2} \text{ ,}$$

where $R_{compr.}(i)$, $R_{orig.}(i)$ – are the coordinates of the contour points of the object, and L is the total number of contour points.

In this case, 20 points were selected, 10 of which were analyzed for displacement in the x x x-coordinate, and the other 10 for displacement in the y y y-coordinate (fig. 1).

In this case, 20 points were selected, 10 of which were analyzed for displacement in the x -coordinate, and the other 10 for displacement in the y -coordinate (fig. 1).

Conclusions and Future Research Directions. This study explored wavelet-based compression methods for video images in medical information-measuring systems, including forward, inverse, and two-dimensional wavelet transforms. Experimental studies on wavelet transforms and compression of digital medical video images showed that at a 10:1 compression ratio, amplitude errors in the video signal are nearly imperceptible. A compression ratio of 50:1 is considered acceptable.

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– надійність автотранспортних засобів.

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Застосування вейвлет-перетворення до стиснення зображень в інформаційно-вимірювальних системах медичного застосування

В сучасних інформаційно-вимірювальних системах (ІВС) найбільш інформативним видом даних є зображення. Це можуть бути зображення, що характеризують стан пацієнтів, а система буде мати медичне застосування. Зображення можуть бути сформовані у видимому діапазоні хвиль електромагнітного випромінювання, або можуть бути результатом фіксації інших видів випромінювання. Велика інформативність як важлива перевага зображень приводить до необхідності передавати та зберігати великий обсяг цифрових даних. Тому для ефективного використання ресурсів комп'ютеризованих ІВС необхідно зменшити цей обсяг шляхом стиснення. Необхідна кількість разів стиснення становить від декількох десятків до сотні разів. Розглянуто теоретичні основи прямого та оберненого вейвлет-перетворення та його застосування у процедурах стиснення зображень медичного застосування, що відбувається з деякою втратою частки інформації. Керуючи процедурою стиснення та обираючи її параметри, можливо забезпечити прийнятну похибку відновлення зображень.

Визначено, що стиснення на основі вибраного типу вейвлета можливе, коли він є ортогональним та відповідно існує обернене вейвлет-перетворення. Також для підвищення швидкодії процедур стиснення запропоновано розділити вейвлет-перетворення на дві одновимірні процедури, що застосовуються до рядків та стовбців зображення. Розглянуто приклад впливу вейвлет-стиснення у форматі JPEG-2000 на точність передачі інформації про геометричні параметри та координати контурних точок об'єктів у медичних застосуваннях зображень у ІВС.

Ключові слова: інформаційно-вимірювальна система; медичні діагностичні зображення; відеозображення; стиснення з частковою втратою інформації; вейвлет-перетворення.

Стаття надійшла до редакції 28.03.2025.