UDC 519.85
DOI: https://doi.org/10.26642/ten-2020-1(85)-155-157

S.I. Yaremchuk, PhD of physico-mathematical science, prof.
O.A. Shupikov, PhD student
«Zhytomyr Polytechnic» State University

Modification of the random search method

The paper considers linear programming problems with Boolean variables. This is a part of mathematical programming focused on solving practical optimization problems, which could be solved and correctly described with a mathematical model of the problem of linear programming with Boolean variables.

The discussed methods for exact and approximate solutions are constructed taking into account the features of these problems. The main focus of the article is the method of the random search. The main idea of this method is formulated and steps of the iterative process are described.

Proposed modification of this method removes the conditions of non-negativity imposed on all coefficients of the problem. This modification makes it possible to apply the method for solving of a much larger number of linear programming problems with Boolean variables.

Keywords: *optimization problem; linear programming; Boolean variables; objective function; optimal solution; system of inequalities; modification.*

A large amount of tasks that arise in various areas of human activity are optimization problems. Quite often their mathematical model is the integer or partial integer linear programming problem. A special case of such problems is the linear programming problem with Boolean variables. One of the examples is the problem of placing the sources of the physical field in fixed places in some area, in which the specified field characteristics in this area would reach the required values. One of the tasks for this example is to place heat sources of different intensity on the fixed places of the area so that the temperature at some specified point of this area would be minimal. Another example is the task of appointing contractors to jobs so that the total cost of these appointments is the lowest. In this case, the number of contractors coincides with the number of jobs, the cost of appointing each contractor to any job is known, jobs and contractors are in a mutually unambiguous ratio. Linear integer programming methods can be used to solve linear programming problems with Boolean variables. But usually the methods built taking into account the features of these tasks are more effective. The following methods are an example of such:

- The additive algorithm [1], which belongs to the group of methods of branches and boundaries (solving linear programming problems with Boolean variables)
- The «P-algorithm» method [2] (solving minimax linear programming problems with Boolean variables)
- Implementation of the general scheme of the decay vector for solving linear programming problems with Boolean variables [3]
- The random search method [4] (solving linear programming problems with Boolean variables)

Formulation of the problem

$$f(x) = \sum_{j=1}^{n} c_j x_j \to \max; \qquad (1)$$

$$\sum_{j=1}^{n} a_{ij} x_j \le b_i, i = 1, \dots, m; \qquad (2)$$

$$x_j = \begin{cases} 0 \\ 1 \end{cases}, j = 1, 2, \dots, n. \qquad (3)$$

All coefficients are assumed to be non-negative.

The main idea of the random search method

The main idea is to replace the solution of the optimization problem (1)–(3) with an iterative process of solving a sequence of systems of linear inequalities

$$\begin{cases} \sum_{j=1}^{n} c_j x_j \ge b_0, \\ \sum_{j=1}^{n} a_{ij} x_j \le b_i, i = 1,2, \dots, m; \end{cases} \qquad (4)$$
$$x_j = \begin{cases} 0 \\ 1 \end{cases}, j = 1,2, \dots, n,$$

where the starting b_0 is chosen based on some considerations.

Each subsequent system (4) is obtained from the previous one by increasing the parameter b_0. The process stops when for a given number of iterations the solution of system (4) could not be obtained.

Iterative solution process

The starting point $x^0(x_1^0, \dots, x_n^0)$ is chosen arbitrarily. At each step k, the corresponding point $x^k(x_1^k, x_2^k, \dots, x_n^k)$, k = 0, 1,.. s obtained and the following expressions are calculated:

$$\Delta_0 = \max\left\{\frac{b_0 - \sum_{j=1}^{n} c_j x_j^k}{b_0}, 0\right\}; \qquad (5)$$

$$\Delta_i = \max\left\{\frac{\sum_{j=1}^{n} a_{ij} x_j^k - b_i}{b_i}, 0\right\}, i = 1, 2, \dots, m. \qquad (6)$$

If all $\Delta_0 = 0(i = 0, 1, \dots, m)$, when the process stops because the solution of system (4) is found. Then b_0 is increased and the new system thus obtained is solved.

If at least one $\Delta_0, i \in \{0, 1, \dots, m\}$ is not equal to zero, then coordinates of the point x^k change and the point x^{k+1} is obtained randomly with the probability

$$p = \min\left\{r, \max_{i \in [0:\, m]} \Delta_i\right\}, r \in (0; 1). \quad (7)$$

This paper proposes a modification of the Random Search Method, which, from the point of view of the authors, will significantly expand the scope of the method and slightly increase the efficiency of its work. The basic idea of the random search method remains. But the conditions of non-negative coefficients in (1)– (3) change, as they significantly narrow the scope of this method. Therefore, the following changes are taking place:

1. Due to the fact that the requirement of non-negativity of the coefficients of the objective function is fundamental to the functioning of the method, the presence negative values among the variables ($c_r < 0, r \in \{1, 2, \dots, n\}$) are being replaced with $\acute{x}_r = 1 - x_r$. Then the objective function takes the form of the sum of the products of non-negative coefficients for the corresponding variables and the negative constant (the constant is not involved in the further solution process).

2. An iterative process of solving the system of inequalities. The starting point $x^0(x_1^0, \dots, x_n^0)$ is chosen arbitrarily. At each iteration of *k* the point $x^k(x_1^k, x_2^k, \dots, x_n^k)$, *k* = 0,1,.. is obtained and the values are calculated (let's call them estimates)

$$\Delta_0 = b_0 - \sum_{j=1}^{n} c_j x_j^k; \quad (8)$$

$$\Delta_i = \sum_{j=1}^{n} a_{ij} x_j^k - b_i, i = 1, 2, \dots, m. \quad (9)$$

If there are no positive estimates, the process is stopped because a system solution is found (4). Then b_0 is increased and the new system thus obtained is solved (4).

If at least one estimate $\Delta_i^k, i \in \{0,1, \dots, m\}$, is greater than zero, then we randomly select the coordinate of the point x^k which is to be changed and a new point x^{k+1} is obtained, and the next k + 1 iteration is performed.

Note. You can randomly select multiple coordinates of a point x^k to be changed. The absence of conditions for the non-negativity of the coefficients in problem (1)–(3) makes it impossible to use formulas (5)–(6) to find estimates. But formulas (8)–(9) for finding estimates are much simpler than (5)–(6). In addition, the absence of these conditions of non-negativity makes it impossible to change the coordinates of the point by the algorithm [4], because formula (7) then does not make sense

Let the initial problem have the form

$$f(\acute{x}) = \sum_{j=1}^{n} c_j \acute{x}_j \to exstr, \quad (10)$$

$$\begin{cases} \sum_{j=1}^{n} a_{ij} \acute{x}_j \le b_i, i = 1, 2, \dots, l, \\ \sum_{j=1}^{n} a_{ij} \acute{x}_j \ge b_i, i = l + 1, \dots, m, \end{cases} \quad (11)$$

$$x_j = \begin{cases} 0 \\ 1 \end{cases}, j = 1,2, \dots, n \quad (12)$$

As mentioned, it can be solved by a modified random search method.

Algorithm of the modified method of random search

1. Problem (10)–(12) is reduced to a form convenient for the application of this modified method, i.e. to the form (1)–(3) with non-negative coefficients of the objective function. To do this, follow these steps.

- If the objective function of the original problem is minimized, then it is multiplied by -1 and takes the form (1).
- If there are negative values among the coefficients of the objective function (1) then the variables are replaced in the problem

$$\acute{x}_j = \begin{cases} 1 - x_j \; if \; j \in J^- \\ x_j, if \; j \; \in J \backslash J^- \end{cases},$$

where $J = \{1, 2, \dots, n\}, J^- = \{j \in J: c_j < 0\}$.

Then the objective function takes the form

$$\hat{f}(x) = \sum_{j=1}^{n} c_j x_j + c \text{, де } c = \sum_{j \in J^-} c_j.$$

The constant *c* is discarded. The function turns out

$$\hat{f}(x) = \sum_{j=1}^{n} c_j x_j \to \max.$$

2. If the constraints include inequalities of type «≥», then both parts of each of them are multiplied by -1 and the system takes the form (2). Thus the problem has the form (1)–(3). Assigned: $\bar{f}(\bar{x}) = -\infty$, where $\bar{x}$ – candidate for the optimal solution. The selected stop criterion is the number of failed iterations in the cycle to find $\bar{x}$ is equal to the limit value N (k = N); or the time T allocated to the solution of the problem is exhausted (t = T).

3. A system of inequalities is constructed and reduced to a special form

$$\begin{cases} \sum_{j=1}^{n} c_j x_j \geq b_0 \\ \sum_{j=1}^{n} a_{ij} x_j \leq b_i, i = 1, 2, \ldots, m \end{cases} \Leftrightarrow \begin{cases} b_0 - \sum_{j=1}^{n} c_j x_j \leq 0 \\ \sum_{j=1}^{n} a_{ij} x_j - b_i \leq 0, i = 1, 2, \ldots, m. \end{cases}$$

The formulas of the corresponding estimations are written down

$$\Delta_0 = b_0 - \sum_{j=1}^{n} c_j x_j ;$$

$$\Delta_i = \sum_{j=1}^{n} a_{ij} x_j - b_i, i = 1, 2, \ldots, m.$$

4. Determine the initial value of $b_0 (b_0 = \min_{j \in [1:n]} c_j)$
5. The system of inequalities constructed in p.2 is solved.
1) The point x^0 is chosen arbitrarily
2) For the point $x^k, k = 0, 1, \ldots$ estimates are calculated

$$\Delta_0^k = b_0 - \sum_{j=1}^{n} c_j x_j^k ;$$

$$\Delta_i^k = \sum_{j=1}^{n} a_{ij} x_j^k - b_i, i = 1, 2, \ldots, m.$$

If a positive estimation is obtained, proceed to p. 4.4., otherwise, the next point is fulfilled.

3) The obtained point x^k is the solution of the corresponding system of inequalities. Proceed to p. 5.
4) The fulfillment of the stop criterion is checked. If it's done – proceed to p. 7. Otherwise - the next step.
5) Randomly select the coordinate number of the point x^k that will be changed. A new point x^{k+1} is obtained, *k* is assigned the value *k* + 1 and proceed to p. 4.2.

6. $\bar{x} = x^k, \bar{f}(\bar{x}) = \bar{f}(x^k)$ (candidate for the optimal solution).
7. $b_0 = \bar{f}(\bar{x}) + \gamma$, де $\gamma = \frac{1}{2}\min\{1, \min_{i \in [1:n]} c_i\}$, proceed to p. 4.
8. The process stops. The optimal solution for a given number of iterations is obtained (given the time allotted for solving the problem. If $\bar{f}(\bar{x}) = -\infty$, then the set of admissible solutions is empty. Otherwise, the specified solution of the problem of the form (1)–(3) has the form $x^* = \bar{x}, \bar{f}(x^*) = \bar{f}(\bar{x})$. There is a corresponding solution of the original problem

$$f(\acute{x}^*) = \begin{cases} \bar{f}(x^*) - c, if\ in\ the\ original\ problem\ f(\acute{x}) \to \max \\ -(\bar{f}(x^*) - c), if\ in\ the\ original\ problem\ f(\acute{x}) \to \min \end{cases},$$

and $\acute{x}^*$.

Conclusions. The problem of mathematical programming which is a problem of linear programming with Boolean variables is considered. Examples of relevant practical problems and methods of their solution are given. The usage of the method of a random search for solving these problems is considered separately. A modification of this method is built. This modification significantly expanded its scope and simplified the corresponding iterative process of solving the system of inequalities. The algorithm of the modified method of random search is given.

References:

1. Balaz, E. (1965), «An additive algorithm for solving linear programs with zero-one variables», *Operats.Res.*, No. 4, pp. 517–546.
2. Yaremchuk, S., Burda, R. and Morgalyuk, O. (2010), «The minimax problem solving of physical fields sources allocation in specified positions », *Electronic Notes in Discrete Mathematics*, pp. 599–606.
3. Sergienko, N.M., Lebedeva, T.T. and Roshin, V.A. (1980), *Priblizhennie metodu reshenia diskretnuh zadach optimizacii*, Naukova Dumka, Kyiv, p. 272.
4. Pyateckiy-Shopiro, I.N., Volkonskij, V.A., Levina, L.V. and Pomanskij, A. (1966), «Ob odnom iterativnom metode reshenia zadach celochiselnogo programirovania», *DAN USSR*, Vol. 169 , No. 6, pp. 1289–1292.

Yaremchuk Svitlana Ivanivna – PhD in Physical and Mathematical Sciences, professor, professor of the Software Engineering Department, the Faculty of Information and Computer Technology, Zhytomyr Polytechnic State University.

E-mail: svetla.yarem@gmail.com.

Shupikov Oleksii Andriyovych – PhD student of the Software Engineering Department, the Faculty of Information and Computer Technology, Zhytomyr Polytechnic State University.

E-mail: shupikovalexey@gmail.com.

The article was sent to the editorial board on 25.03.2020.