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An algorithm for detecting the minimum allowable accelerometer sample rate for tracing translational motion along a Bézier curve

Inertial sensors are used for human motion capture in a wide range of applications. Some kinds of human motion can be tracked using inertial sensors incorporated in smartphones or smartwatches. However, the latter can scarcely be used if misclassification of user activities is intolerable. In this case electronics and embedded software engineers should design, implement and verify their own human motion capture embedded systems, and oftentimes they have to do so from scratch. One of the issues the engineers should face is selection of suitable components, including accelerometers, after thorough examination of commercially available items. Among technical characteristics of accelerometers their sample rate determines whether the sensor will be able to capture a specific motion kind or not. The paper introduces an algorithm for calculating the minimal accelerometer sample rate sufficient for tracking a prescribed translational motion scenario without significant signal losses. The algorithm is based on the equations of velocity and acceleration vectors, Bézier curves and their derivatives and signal frequency analysis. The proposed algorithm takes a few control points and a time period spent on travelling along a curve. The points and time period are input by the user. The algorithm constructs a smooth Bézier curve or a set of Bézier curves pieced together seamlessly. Then it simulates a signal that might have come from the ideal accelerometer if it could have been used for tracking the curve. A suitable sample rate is chosen iteratively, starting from the minimal sample rate claimed sufficient for human motion capture, by calculation a root mean square deviation for the reference signal and a signal sampled at the current rate. The algorithm is proven to be applicable and is to be integrated into a larger system for assessment of inertial sensors.

Keywords: *sample rate; translational motion; MEMS accelerometer; Bézier curve; acceleration vector.*

Introduction. Modern inertial sensors manufactured using MEMS (Micro-Electro-Mechanical Systems) technology have numerous advantages, including small size, light weight and low power consumption, which makes them wearable. Moreover, such sensors are self-containing, i.e., one does not need any external tools for tracking the position and attitude of an object supplied with such sensors, both indoors and outdoors. The latter property coupled with the possibility to embed MEMS inertial sensors into clothes and footwear gives rise to a variety of applications, primarily in human motion capture systems. Constant tracking motions and position of aging or convalescent people in their natural living environment helps understand the cause of many diseases and thus provides valuable information for medical diagnosis and/or feedback on the treatment efficacy. Among the common neurological problems that affect human motor functions are Parkinson disease, multiple sclerosis, stroke, traumatic brain injury, and spinal cord injuries [1]. If a patient is supposed to perform exercises on their own, a human motion capture system can replace surveillance from medical personnel. Fall detection systems are another popular research subject, since they can mitigate the fall consequences by reporting a fall straightaway after it occurred and calling for immediate medical help. Fall prevention is even more challenging. Inertial sensors are getting increasingly helpful in analysis of sport activities both for performance assessment and injury prevention. Some sports products such as the Philips DirectLife or the Nike+ running shoes integrate motion sensors and offer both amateur and professional athletes feedback on their performance [2]. Some researchers utilize inertial sensors for handwriting recognition. Another vast application area relates to classification of an employee's movements as correct or incorrect. For instance, assembly workers or nurses can be subjected for such an analysis. A lot of recent research results have been published on gait analysis and classification of human activities such as walking, ascending or descending, cycling, running, swimming, standing, sitting and lying. Activity recognition has been widely used in game consoles such as the Nintendo Wii. In virtual reality applications, computer graphics systems use movement analysis to provide position and orientation control in a head-mounted display [1]. Besides, human motion capture is used in human-machine interaction, user authentication [3], robotics and telemedicine [4]. The appearance of MEMS inertial sensors became a decent means to overcome the inherent limitations of other motion capture systems – optical, structured light, acoustic, magnetic and mechanical ones. Commercial optical systems such as Vicon or Optotrack are considered to be the gold standard in human motion capture due to their high accuracy. Optical motion capture systems incorporate dozens of digital cameras and hundreds of active or passive reflective markers attached to different body segments. First, the cameras capture the position of the markers, then the position and attitude of the body are computed using various algorithms. As one can infer from the operation principle of optical systems, they are rather costly and can only be used in a laboratory environment. Moreover, there should be a clear line of sight between the body and the cameras, i.e., optical systems suffer from occlusions, and so do structured light and acoustic systems. Acoustic motion capture systems are based on emitters and receivers of sound waves. The former sends out ultrasonic pulses whereas the latter catches the pulses. The distance between the emitter and receiver is calculated upon the time period it takes for the pulses to reach the receiver. Along with occlusions, humidity and temperature fluctuations also impair acoustic systems. Magnetic motion capture systems rely on magnetic sensors to measure artificially generated magnetic fields. They are susceptible to noise and electromagnetic disturbances and can only be used in environments where such disturbances have been eliminated. Mechanical systems incorporate a chain of metal/plastic items coupled with electromechanical transducers. They are highly precise, however, can be too bulky and inconvenient to monitor daily activities [1]. The benefits of inertial sensors are corroborated by the variety of commercially available inertial measurement units (IMUs) and IMU-based systems manufactured by Invensense and Microstrain (the USA), Trivisio (Germany), and XSens (The Netherlands) [5].

However, MEMS inertial sensors have their limitations [6], and one should thoroughly examine their hardware characteristics including sample frequency before design a motion tracking system.

Related work. Human motion capture problems are widely covered in the scientific literature. One can distinguish several main research directions. A large research area concerns usage of smartphones for recognition of their owners' basic daily activities. Modern smartphones are supplied with various sensors such as 3D accelerometers and magnetometers, compasses, proximity sensors, GPS, digital cameras and microphones. Such equipment coupled with the omnipresence of smartphones make the latter useful for scientific and clinical research, for instance in healthcare and physical activity monitoring. Ref. [7] studied the applicability of smartphones for clinical motion research. The authors showed that smartphones could be used to assess range of motion and joint angle measurement for postural and gait control. They benchmarked performance of different smartphone sensors against an XSens product. Ref. [8] proposed a human fall monitoring system comprised by a portable sensor unit including a 3D accelerometer, a 3D gyroscope and a 3D magnetometer and a cell phone.

Smartphones are not an option in industrial context, where occasional misclassifications of human activities are intolerable. Thus a plenty of specially designed devices appeared. In contrast to smartphones, such devices can afford more complicated recognition algorithms because they are not that restricted in battery consumption and computational complexity. One of the main research topics in this case is the required amount and location of sensors and suitable recognition methods depending on complexity, periodicity, and dynamicity of activities to be analyzed. Ref. [9] introduced three strategies to measure motions of classical cross-country skiing, ski mountaineering, alpine ski racing and outdoor walking over long distances. These strategies found an articulate implementation in a system for alpine ski racing, which is highly dynamic, i.e., characterized by fast direction changes, high speeds and the absence of static or slow phases. The proposed methods rely only on inertial sensors and magnetometers. Nevertheless, they provide position, attitude and speed information with an accuracy close to the «gold standards». For each activity specific biomechanical constraints and movement dynamics were exploited. The authors emphasize the role of the sample frequency. They pinpoint two error types for signal sampling: signal losses caused by an insufficiently high sampling frequency and inadequate low-pass filters at analogue-to-digital conversion. Signal losses are especially likely to occur for rapidly changing movements such as foot movements during gait. The IMU's sampling frequency used in [9] was 500 Hz, which the authors found to be sufficient for their sensors to measure all movements accurately. Ref. [10] discussed fusion of GNSS with data from inertial and magnetic sensors in order to analyze performance in alpine ski racing. The authors sampled magnetometer data at 125 Hz and data from 3D accelerometers and gyroscopes at 500 Hz. The same sample frequencies were used in the experimental setup of [11]. Ref. [12], among other valuable dissertation results, discusses the influence of accelerometer sample frequency on the recognition rate when classifying different swimming styles (two different sensor locations were considered). Besides, in order to reduce battery consumption, the researcher exploits the characteristics of long-term activities: if one was cycling for a while, it is most likely that the same person will be still cycling at the next moment. In his thesis [12] Pekka Siirtola took an original 50 Hz signal, formed 5 Hz, 10 Hz and 25 Hz signals by picking each 10^{th}, 5^{th} or 2^{nd} point correspondingly and recorded the recognition rate of three periodic activities – freestyle, breaststroke and backstroke swimming – for each sample frequency. He revealed that the recognition rate did not suffer much from the sample frequency reduction.

According to [13], a sample frequency 100 Hz is enough for capturing active human motion in sufficient detail. Ref. [14] presented an implementation of a system for 3D attitude measurement and estimation using a magnetic and inertial measurement unit and a Kalman-filter based sensor fusion algorithm. They stated the used sample frequency 200 Hz. The researchers benchmarked their results against traditional optical motion capture systems. Ref. [15] investigated the impact of the amount, location and type of inertial sensors on the accuracy of activity and posture recognition. The authors used XSens-MTx sensors, each equipped with a 3-axis accelerometer, 3D gyroscope and 3D magnetometer, with the sampling frequency 6 Hz. According to [16] and [17] sampling frequencies exceeding 50 Hz should be used. However, in [18] experiments at the frequency of 2 Hz proved to be successful – eleven activities were distinguished with the average recognition rate of 85 %.

Ref. [19] studied the influence of different sampling rates on the recognition accuracy of ten daily activities. The research results prove that the reduction of sampling frequency from 100 Hz to 5 Hz affects different activities in a different way. The worst effect was observed for a walking activity. Ref. [20] introduced a system that utilizes a wearable device and an effective quaternion algorithm for timely fall detection. The system distinguishes a fall from normal everyday activities and alarms the caregivers. In accordance with [20] 100 Hz is a proper sample frequency for human fall detection.

Ref. [21] deals with a choice of a MEMS accelerometer suitable for a specific application. However, the author does not take into account a motion scenario to be captured. Ref. [22] studied the applicability of some IMUs for tracking specific motion kinds. The issues stem from the fact that the static and dynamic accuracy of an IMU dictate whether or not the IMU is suitable for a specific application, and the manufacturer of an inertial sensor often does not specify for which motions their dynamic accuracy specification is valid.

Literature study shows that most researchers focus mainly on mathematical processing of inertial sensor data. Primarily they modify, adapt, adjust and apply data fusion algorithms (Kalman, extended Kalman, unscented Kalman, particle filters, Madgwick filter and others) or activity classification algorithms (linear and quadratic discrimination analysis, naïve Bayes, J48, Random forest, etc). Relatively few research works consider the characteristics of the underlying inertial sensors including their sample frequencies. Such works analyze the effect of the sample frequency on the motion capture (for instance, on human activity recognition rate). However, they do not try to figure out which sample frequency would be enough for successful motion capture and do not establish dependencies between sample frequencies and expected motion trajectories.

Nevertheless, an insufficient sampling frequency means signal losses that can scarcely be compensated for by pure mathematical approaches.

The aim of the work is providing a mathematical background and developing an algorithm for discovering the minimal accelerometer sample frequency, sufficient for tracking the translational part of an expected motion scenario described by Bézier curves.

Mathematical background and algorithm description. Any motion of a rigid body can be represented as a combination of translational and rotational motions, each of them considered independently. The former can be characterized by readings of a triaxial accelerometer.

A triaxial accelerometer measures the projections of the apparent acceleration vector onto the axes assigned to the device. That is, the gravity vector is supposed to be eliminated from the readings, leaving the user with «true» accelerations. Such elimination is an issue itself. However, we do not consider acceleration due to gravity in our work. As is well known, the acceleration vector is the derivative of the velocity vector with respect to time:

$$\bar{a} = \lim_{\Delta t \to 0} \frac{\Delta \bar{v}}{\Delta t} = \frac{d\bar{v}}{dt} \tag{1}$$

The velocity vector is defined as:

$$\bar{v} = \lim_{\Delta t \to 0} \frac{\Delta \bar{r}}{\Delta t} = \frac{d\bar{r}}{dt}, \tag{2}$$

where $\bar{r}$ is the position vector of an object whose velocity is being estimated. A position vector (also known as location vector or radius vector) is a vector that represents the position of an arbitrary point in the Euclidian space in relation to some reference point. The latter is typically the origin of the chosen coordinate system. In the Cartesian coordinate system the position vector is expressed as:

$$\bar{r} = x(t)i + y(t)j + z(t)k, \tag{3}$$

where t is a time parameter. Thus, the acceleration vector can be rewritten as:

$$\vec{a} = \frac{\partial^2 x(t)}{\partial t^2} i + \frac{\partial^2 y(t)}{\partial t^2} j + \frac{\partial^2 z(t)}{\partial t^2} k. \tag{4}$$

Since translational and rotational movements can be considered independently and angular velocities are measured by gyroscopes, we can ignore any rotational movements when assessing the applicability of an accelerometer. Thus, in the case of accelerometers we are interested in a way to construct smooth spatial curves built upon a reasonable amount of control points. Obviously, an embedded software or electronics engineer will appreciate a user-friendly visual tool that allows adjusting curves by dragging their control points.

A well-known mathematical instrument for this task is Bézier curves.

A Bézier curve is a parametric curve defined by its control points, P_0, …, P_m. Number m is the Bézier curve order ($m = 1$ for linear Bézier curves, $m = 2$ for quadratic ones, etc.). If all the control points are known, the curve can be found using the following equation:

$$p(t)=\sum_{i=0}^{m} B_i^m(t)P_i, \quad t\in[0,1] \tag{5}$$

Thus, a Bézier curve is a linear combination of Bernstein polynomials, which are defined as:

$$B_i^m(t)=\frac{m!}{i!(m-1)!}t^i(1-t)^{m-1} \tag{6}$$

Bernstein polynomials are always non-negative in the interval $[0,1]$.

Function (5) can be split into three functions for description of the *x*, *y* and *z* coordinates. The forms of the three functions are the same. The only difference is that one function uses only *x*-coordinates of the control points, whereas the others utilize only *y* or *z* coordinates. For example, if one deals with a cubic Bézier curve, the curve is defined by these three equations:

$$x(t)=P_{x0}(1-t)^3+3P_{x1}t(1-t)^2+3P_{x2}t^2(1-t)+P_{x3}t^3, \tag{7}$$

$$y(t)=P_{y0}(1-t)^3+3P_{y1}t(1-t)^2+3P_{y2}t^2(1-t)+P_{y3}t^3, \tag{8}$$

$$z(t)=P_{z0}(1-t)^3+3P_{z1}t(1-t)^2+3P_{z2}t^2(1-t)+P_{z3}t^3. \tag{9}$$

Using the Bézier curve equation in conjunction with the properties of Bernstein coefficients, one obtains the derivative of the Bézier curve:

$$p'(t)=\sum_{i=0}^{m-1} B_i^{m-1}(t)(P_{i+1}-P_i). \tag{10}$$

It is possible to construct a Bézier curve with multiple control points. However, the more control points are involved, the higher the order of the curve is, which means more complicated computations. In this case it is better to split a single Bézier curve into several low-order curves. Each subcurve is a Bézier curve as well. One has to piece low-order curves together seamlessly. Such seamlessness can be achieved if all the consecutive component curves share one end point and have the same curvature in this point. The curvature can be calculated as:

$$\kappa=\frac{\left((z''y'-y''z')^2+(x''z'-z''x')^2+(y''x'-x''y')^2\right)^{1/2}}{\left(x'^2+y'^2+z'^2\right)^{3/2}} \tag{11}$$

The first and last control points (P_0 and P_m correspondingly) are always the end points of the curve. Interim points typically do not belong to the curve and just serve for its motion control. Not every curve can be approximated easily by Bézier curves. Surprisingly enough, a circle can only be rather accurately approximated by a four-piece cubic Bézier curve.

Since Bézier curves provide a parametric description of all the space coordinates independently, they become a natural choice for representation of a motion trajectory to be tracked. However, the time parameter can only run the values from 0 to 1. This fact should be taken into account if one needs to deal with different time intervals.

In accordance with the well-known Shannon-Nyquist theorem, the sampling frequency for a signal should not be less than the doubled maximum frequency present in this signal. The minimum allowed sample frequency is called the Nyquist frequency. It is considered a sufficient frequency for reconstruction of the original signal without significant losses, which does not necessarily mean a required frequency.

In order to perform a signal analysis in the frequency domain, the Fourier transform and its modifications are usually used.

The Fourier transform decomposes an arbitrary signal existing for the whole time axis into a sum of sinusoids of different frequencies. The Fourier transform $\hat{f}(w)$ of function $f(x)$ and its inverse are computed as follows:

$$\hat{f}(w)=\int_{-\infty}^{\infty} f(x)e^{-2\pi ixw}dx \quad ; \tag{12}$$

$$f(x)=\int_{-\infty}^{\infty} \hat{f}(w)e^{2\pi iwx}dw \quad . \tag{13}$$

Graphically, the Fourier transform can be shown as a diagram that displays the amplitude and frequency of each of the determined sinusoids.

If a signal to be analyzed in the frequency domain has been already subjected to time-quantization, then it is processed using the discrete-time Fourier transform (DTFT). If we have a signal only defined for some finite interval $[t_1,t_2]$ instead of the whole time axis, we assume the signal has a zero value on $[-\infty,t_1)$ and $(t_2,\infty]$, which can be achieved by applying a window function. The latter leads to spectral leakages, however. The output of the DTFT is continuous in frequency and periodic, which makes it inconvenient for automated processing.

In order to use computers for calculating the frequency spectrum of a discrete-time signal, the discrete Fourier transform (DFT) is used. The DFT is defined as:

$$Y(k)=\sum_{j=1}^{N} X(j) W_N^{(j-1)(k-1)}, \quad W_N = e^{\frac{-2\pi i}{N}} . \tag{14}$$

The DFT is an invertible transform, whose inverse is calculated as:

$$X(j)=\frac{1}{N}\sum_{k=1}^{N} Y(k) W_N^{-(j-1)(k-1)} . \tag{15}$$

The fast Fourier transform (FFT) efficiently computes the DFT, whose computational complexity is $O(n^2)$. The FFT reduces the computational complexity to $O(n \log n)$.

In order to define the minimum accelerometer sample frequency suitable for tracking motion along a Bézier curve, we propose to piece together the mathematical apparatus of acceleration and velocity, Bézier curves, and frequency analysis.

Our algorithm is comprised by the following steps.

Step 1. Prescribing a Bézier curve. As has been mentioned before, the user, who is an embedded software or electronics engineer, should be given a possibility to set only a few control points upon which a Bézier curve will be built. For flexibility considerations, there should be a possibility to set a changeable amount of points, since the user may need to trace simple movements (like waving a hand, for instance) or much more complicated or highly dynamic ones (nursing activity, cooking, dancing or skiing). A Bézier curve may be built in several ways. The most straightforward way is to construct a curve of order (*N*–1) provided that the user has set *N* control points. However, this approach consumes a lot of computational resources, mainly due to the fact that Bernstein coefficients assume calculation of factorials. This resource consumption can be reduced by usage of look-up tables, i.e., any interim factorial calculations can be saved and fetched again rather than computed from scratch. Look-up tables are a trade-off between memory consumption and execution time. An alternative way is to build a set of cubic, quartic or uantic Bézier curves upon groups of four consecutive control points so that these curves form a composite Bézier curve. The latter should look smooth and seamless as if it were a single curve, not a collection of smaller curves. As has been mentioned before, such an approach requires the curvature (11) to be calculated. The obtained Bézier curve itself should be visualized only with the purpose to offer the user a motion trajectory and get their approval. Further calculations involve the derivatives of Bézier curves instead of the curves themselves. It may happen that the curve built automatically does not meet the user's expectations. In this case, the user will require the possibility to drag the control points manually and thus adjust the curve in the manner of professional graphical software. Since visualization quality has little influence on the results of accelerometer assessment, there is freedom in drawing Bézier curves. Moreover, the time step does not directly affect the derivatives, since there exist pure analytical functions for the latter. The main result of Step 1 is the equations for the Bézier curve or the set of curves, whose suitability has been approved by the user.

Step 2. Calculation of the acceleration vector. Using equation (4), we calculate the second derivative with time of the three coordinates of the spatial Bézier curve obtained at the previous step. The quantization step of the derivatives does not depend on the quantization step chosen previously for visualization of the Bézier curve(s). If there is only one Bézier curve, then the time parameter is within the range $[0,1]$. Otherwise, the total time interval for all the component curves is defined by their number. The result of Step 2 is an array *A* of the vectors that describe acceleration for each time moment corresponding to the chosen quantization step.

Step 3. Time mapping. Along with the control points, the user should be given a possibility to preset the time period, spent on moving along the prescribed trajectory. Thus, the time parameter used for Bézier curve construction should be mapped on the real, physical time. Bézier curves are non-linear; they demonstrate different distances travelled during the same time intervals. The time points themselves are distributed evenly during construction of a Bézier curve. Therefore, the only thing one needs to do is to scale the time period, i.e. either shrink or expand it. Consequently, faster movements mean shorter time periods and more acquired points per a time unit. Therefore, faster movements require higher sample frequencies. After mapping, array *A* contains acceleration vectors corresponding to modified time moments. The result of Step 3 is an emulated signal of an ideal accelerometer, which does not suffer from any noise, quantization errors or insufficient sample frequency. Such a signal might have been obtained when tracking a prescribed trajectory if only the real accelerometer could have been free of the mentioned problems. It is worth bearing in mind that array *A* represents acceleration expressed in the inertial frame whereas accelerometers provide data in respect to their body frame. Thus, in general, coordinate transformation is required. However, coordinate translation will not change the spectral characteristics of the signal whereas rotation is out of the scope of this paper.

Step 4. Extracting three components of the acceleration vector. For convenience of future analysis, we decompose array *A* into three vectors, A_X, A_Y and A_Z, which represent measurements along *x*, *y* and *z* axis of the ideal accelerometer, correspondingly.

Step 5. Frequency analysis of the acceleration vector components. Taking into consideration the above-stated properties of the FFT and its inverse, a suitable sample frequency will be determined iteratively as follows. First, we start with some low initial sample frequency Fs, which is still sufficient for human motion capture. We start with a sample rate that has been fetched out of our literature study. The reference discrete-time signal A_X, which simulates the readings of the ideal accelerometer along the axis *x*, is subjected to the FFT at sample frequency Fs_X. The obtained result is transformed back into the time domain using the inverse FFT. Thus we obtain some discrete-time signal A_X^*, which should reproduce A_X accurately if the sample rate has been chosen correctly. The correctness can be characterized by the discrepancies between the reference signal and A_X. In order to estimate the discrepancies, signal A_X^* should undergo the linear interpolation in order to fill up the missing readings and thus equalize the lengths of the two signals for further comparison. Then the two signals, the reference one and the linearly interpolated A_X^*, are compared using the root-mean-square deviation (RMSD):

$$RMSD = \frac{\sqrt{\sum_{t=1}^{T}\left(A_{X,t} - A_{X,t}^*\right)^2}}{T} . \tag{16}$$

If the sample rate is sufficient, the RMSD tends to zero. As one can expect, large RMSD values indicate significant discrepancies and, therefore, a deficient sample rate. Some threshold RMSD value should be preset. Step 5 should be taken iteratively until the current sample rate provides such a RMSD value that does not exceed the acceptable threshold. As soon as the preset threshold is achieved, the search is stopped. The current sample rate value is considered sufficient for tracing the prescribed motion along the *x* axis. Two remaining signals, A_Y and A_Z, are processed in the same manner. The result of Step 5 is three numbers, Fs_X, Fs_Y, and Fs_Z. The numbers indicate the minimal sample rate suitable for tracking motion along the three coordinates of the prescribed Bézier curve.

Step 6. Processing the results. The object being tracked can rotate during motion along the Bézier curve. This means that if no coordinate transformation was performed, measurements along different axes of the inertial frame will be mixed. It is not possible to compensate for the rotational movements without auxiliary sensors (gyroscopes and magnetometers). Moreover, even if these two sensors are available, they have their limitations. A gyroscope accumulates measurement noise whereas a magnetometer is susceptible to interferences induced by metal items in vicinity. Therefore, the most efficient solution is to choose the maximum value among the triple Fs_X, Fs_Y, and Fs_Z. It will be the final argument for taking a decision on whether a particular accelerometer is suitable for tracking an object that moves along the specified Bézier curve or not.

Verification of the proposed algorithm has been performed using two different software tools. It is worth mentioning that the work lies within complex research into development of MIS-IDE (Measurement Inertial System Integrated Development Environment) [23] that has been conducted by a team of scholars (including the authors) within a series of commercial projects. MIS-IDE is intended for enhancing the efficacy of synthesizing firmware of navigation systems based on IMU sensors. The scope of the complex work covers developing: models of simulating signals from IMU sensors in accordance with some preset motion scenario, hardware-software tools for testing IMU sensors; algorithms for parametric analysis and synthesis of IMU sensor models, mathematical models for synthesis of noise and measurement errors, algorithms for selection of the most suitable measurement modes, firmware for microprocessors inside embedded systems based on IMU sensors.

At the first step, a number of Bézier curves have been constructed. Since the user is interested in a visual, interactive drag-and-drop tool for building a trajectory, we used our own software module [24]. The module was developed primarily with the purpose to integrate it into MIS-IDE. It is based on OpenGL and allows the user to either select a well-known motion scenario (for instance, a bouncing ball trajectory), or set a customized one. The basic idea of the module is that any embedded software engineer can prescribe a motion curve even if they are not familiar with OpenGL programming. The tool has not been integrated in MIS-IDE yet, however, it allows the user to save all the data into a file.

Then we exported the basic parameters of the Bézier curves into MATLAB and performed the rest of the above-stated algorithm steps.

Despite the fact the authors have a lot of readings from real accelerometers, gyroscopes and magnetometers, none of them was used in the verification procedure. The reason is that in our MIS-IDE we calculate motion trajectories upon readings from these sensors using data fusion algorithms, which are not impeccable themselves, and there is no reference motion tracking systems to ensure the correctness of the calculated trajectories.

Conclusions and future research. The proposed algorithm is applicable for detecting the minimal sample frequency of an accelerometer, which would be sufficient for tracking a prescribed translational motion. The algorithm has been verified using MATLAB and a custom OpenGL module for prescribing motion trajectories. We are planning to integrate it with the complex hardware and software tool for inertial sensors assessment, MIS-IDE.

Currently the proposed algorithm has been verified for a limited amount of control points for Bézier curves. Our future work will be implementation of more complex Béziers and comparison of the ways of constructing high-order Bézier curves.

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